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Influence of Deleting Some of the Inputs and Outputs on Stability Return to Scale in DEA

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ABSTRACT

Data envelopment analysis (DEA) is a nonparametric method for determining the relative efficiency of homogeneous decision making units (DMU) which consist of multiple inputs and outputs. One of the most important issues in DEA is sensitivity analysis and stability of return to scale (RTS) with changing the inputs and outputs. Deleting one or multiple inputs or outputs in DEA can change the efficiency and RTS of some DMUs which is shown by an example. In this paper our aim is to investigate the impact of deleting one or multiple inputs and (or) outputs on RTS and efficiency of DMUs. To this end some models is presented and they are utilized through two examples.

Keywords: DEA, sensitivity and stability, return to scale and optimization.

1. INTRODUCTION

Data Envelopment Analysis is a nonparametric method which was first initiated by Rhodes in PhD thesis to the guidance of Professor Cooper. Their works were published with cooperation of Charnes and Cooper (1978) known as CCR's paper. In fact, it was the generalization of Farrell' work (1957) to multiple inputs and outputs to determine the efficiency of decision making units by using linear programming. Then BCC model that is an extension of CCR model was presented by Banker, Charnes and Cooper (1984). These two papers were base of many studies in the performance analysis which progressed rapidly. One of the most important issues in DEA is to determine the type of RTS. Banker *et al*. (1992) presented models to identify type of returns to scale by using the multiplier form of BCC model. One of the important issues in DEA is sensitivity analysis. At first, sensitivity analysis in DEA was considered by Charnes *et al.* (1985) with changing one output. After that several studies were presented about changes in multiple inputs and (or) outputs, for example Seiford *et al.* (1998), Zhu (2001), Cooper *et al.* (2001, 2007), Jahanshahloo *et al.* (2004, 2005a, 2005b) and etc. In this study, sensitivity analysis is contributed on return to scale and efficiency of DMUs with deleting some of inputs and (or) outputs. To the end, several models are presented for preserving efficiency and RTS of decision making units.

This study consists of the following sections. In Section 2, some basic concepts of DEA and returns to scale are discussed. In section 3, the impact of removing one or multiple inputs and (or) output is investigated on the RTS of DMUs through introducing some models. In Section 4, the stability conditions of RTS and the presented models for removing inputs and (or) outputs are considered through two examples. Finally in Section 5 the conclusions of this study are described.

2. BASIC CONCEPTS

Suppose a set of n decision making units are as DMU_j $(j-1,2,\ldots,n)$ that use m inputs $x_{1j},x_{2j},\ldots,x_{mj}$ to produce s outputs $y_{1j},y_{2j},\ldots,y_{sj}$. The multiplier form of BCC model is as follows:

Model 1- Multiplier form of BCC model

$$\begin{aligned}
&\text{Max} \quad \sum_{r=1}^{s} u_r y_{ro} + u_0 \\
&\text{s.t} \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 \quad j=1,\ldots,n \\
&u_r \ge 0 \qquad r=1,\ldots,s \\
&v_i \ge 0 \qquad i=1,\ldots,s
\end{aligned}$$

where o is the index of evaluated unit, $U=(u_1,u_2,\ldots,u_s)\in R^s$ and $\mathrm{V} = (\mathrm{v}_1, \mathrm{v}_2, \ldots, \mathrm{v}_\mathrm{m}) \in \mathrm{R}^\mathrm{m}$.

Definition 1. DMU_o is called BCC efficient if there is an optimal solution of (model1) with $v_i^* > 0$ for $i = 1, \dots, m$, $u_r^* > 0$ for $r = 1, \dots, s$ and $\sum_{r=1}^{s} u_r^* y_{ro} + u_o^* = 1$. Otherwise it is called BCC inefficient.

Definition 2. DMU_o is called at least weak efficient ($(X_o, Y_o) \in \partial T_v$) if the optimal objective function of Model 1 is equal to one.

Return to scale in data envelopment analysis is defined as the rate of changing output to input that is important for management decisions. To the end different methods has been presented for calculating RTS of DMUs. One of these methods that have been presented by Banker *et al.* (1992) is as follows.

Suppose that $DMU_o \in \partial T_v$ (DMU_o is at least weak efficient) and let (U^*, V^*, u_0^*) as the unique optimal solution of Model 1;

(a) If $u_0^* > 0$ then DMU_o has increasing return to scale.
(b) If $u_0^* < 0$ then DMU_o has decreasing return to scale.
(c) If $u_0^* = 0$ then DMU_o has constant return to scale.

3. IMPACT OF DELETING INPUTS AND OUTPUTS ON RETURN TO SCALE

Suppose that DMU_o has been evaluated before. Now it is reevaluated with deleting one or multiple inputs and (or) outputs to find out the impact of this modifications on efficiency and return to scale status of DMU_o. To this end at first an example with 12 DMUs, 2 inputs and 2 outputs from Cooper *et al*. (2007) is considered (Table 1). Efficiency of these DMUs are obtained by CCR and BCC models. Also type of Return to Scale (only at least weak efficient DMUs) are determined. Then the results of deleting inputs and outputs on efficiency and RTS status of DMUs are presented in Table 2-1 and Table 2-2. As it can be seen, with deleting some inputs and (or) outputs, if the efficiency is preserved, then the type of returns to scale may be changed.

For example, in DMU2 after deleting O1 or I1+O1 or I2+O1 efficiency is preserved but RTS changes from Constant to Increasing. Also in DMU4 after deleting O2 or I1+O2 efficiency is preserved but RTS changes from constant to Deceasing. So we are looking for conditions which with deleting the inputs and outputs of a DMU (at least weak efficient), the type of RTS is preserved. Therefore in this section, the impact of deleting one or multiple inputs and (or) outputs on return to scale is investigated through some models. At First, deleting one input or one output is considered then deleting multiple inputs and (or) outputs will be considered.

TABLE 1: DMU's Data (Cooper *et al*. (2007))

Hospital (DMU)	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Doctors (I1)	20	19	25	27	22	55	33	31	30	50	53	38
Nurses (I2)	151	131	160	168	158	255	235	206	244	268	306	284
Outpatients (O1)	100	150	160	180	94	230	220	152	190	250	260	250
Inpatients (O2)	97	50	55	72	66	90	88	80	100	100	147	120

TABLE 2-1: Efficiency and RTS before and after deleting inputs and outputs

	CCR-eff	BCC-eff	RTS	Deleting I_1		Deleting I_2		Deleting O_1		Deleting O_2	
				BCC-eff	RTS	BCC-eff	RTS	BCC-eff	RTS	BCC-eff	RTS
DMU_1	**1.00**	**1.00**	**CRTS***	1.00	CRTS	1.00	CRTS	1.00	CRTS	0.95	
DMU_2	**1.00**	**1.00**	**CRTS**	1.00	CRTS	1.00	CRTS	1.00	**IRTS****	1.00	CRTS
DMU_3	0.88	0.90		0.90		0.84		0.83		0.90	
DMU_4	**1.00**	**1.00**	**CRTS**	1.00	CRTS	0.92		0.84		1.00	**DRTS**
DMU_5	0.73	0.88		0.87		0.88		0.88		0.86	
DMU_6	0.83	0.94		0.94		0.62		0.58		0.94	
DMU_7	0.90	**1.00**	**DRTS*****	0.96		0.98		0.63		1.00	DRTS
DMU_8	0.78	0.78		0.78		0.74		0.70		0.65	
DMU_9	0.94	0.98		0.85		0.98		0.73		0.89	
DMU_{10}	0.87	**1.00**	**DRTS**	1.00	DRTS	0.76		0.60		1.00	DRTS
DMU_{11}	0.94	**1.00**	**DRTS**	1.00	DRTS	1.00	DRTS	1.00	DRTS	1.00	DRTS
DMU_{12}	0.94	**1.00**	**DRTS**	0.98		1.00	DRTS	0.93		1.00	DRTS

**CRTS=constant return to scale, **IRTS=increasing return to scale, ***DRTS=decreasing return to scale*

TABLE 2-2: Efficiency and RTS before and after deleting inputs and outputs

	CCR-eff	BCC-eff	RTS	Deleting I_1+O_1		Deleting I_1+O_2		Deleting I_2+O_1		Deleting I_2+O_2	
				BCC-eff	RTS	BCC-eff	RTS	BCC-eff	RTS	BCC-eff	RTS
DMU_1	**1.00**	**1.00**	**CRTS**	1.00	CRTS	0.87		1.00	CRTS	0.95	
DMU_2	**1.00**	**1.00**	**CRTS**	1.00	**IRTS**	1.00	CRTS	1.00	**IRTS**	1.00	CRTS
DMU_3	0.88	0.90		0.83		0.90		0.76		0.84	
DMU_4	**1.00**	**1.00**	**CRTS**	0.84		1.00	**DRTS**	0.72		0.92	
DMU_5	0.73	0.88		0.87		0.83		0.88		0.86	
DMU_6	0.83	0.94		0.58		0.94		0.36		0.62	
DMU_7	0.90	**1.00**	**DRTS**	0.63		0.96		0.60		0.98	

DMU_8	0.78	0.78		0.70		0.65		0.63		0.63	

TABLE 2-2 (continued): Efficiency and RTS before and after deleting inputs and outputs

	CCR-eff	BCC-eff	RTS	Deleting I_1+O_1		Deleting I_1+O_2		Deleting I_2+O_1		Deleting I_2+O_2	
				BCC-eff	RTS	BCC-eff	RTS	BCC-eff	RTS	BCC-eff	RTS
DMU_9	0.94	0.98		0.66		0.75		0.73		0.89	
DMU_{10}	0.87	**1.00**	**DRTS**	0.60		1.00	DRTS	0.44		0.76	
DMU_{11}	0.94	**1.00**	**DRTS**	1.00	DRTS	1.00	DRTS	1.00	DRTS	1.00	DRTS
DMU_{12}	0.94	**1.00**	**DRTS**	0.78		0.94		0.93		1.00	DRTS

3.1 Deleting one input or one output

Suppose that $DMU_o \in \partial T_v$ has increasing return to scale. ($u_0^*> 0$). One of these inputs or outputs is removed. With due attention to this point that deleting any input or output is equivalent to deleting a variable in Model 1 and this fact that if the optimal value of a variable is equal to zero then deleting it has no effect on optimality, it can be concluded that deleting any input or output (with v_i^*=0 or u_r^*=0) has no effect on the efficiency of DMU_o. So for recognizing the impact of deletion one input (one output) on DMU_o, it is enough to consider the value of corresponding weight of input (output) in the optimal solution and for preservation increasing return to scale, the constraint of $u_o \geq \varepsilon$ should be added. Therefore, for recognizing the impact of deleting k(th) input for $k \in \{1, 2, \dots, m\}$ on DMU_o (with increasing return to scale) the following model is presented:

Model 2- Preservation IRTS for deleting k(th) input

$$
\begin{aligned}
&\text{Min} \quad v_k \\
&\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\sum_{r=1}^{s} u_r y_{ro} + u_0 = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \leq 0 \qquad j = 1, \dots, n \\
&u_r \geq 0 \qquad r = 1, \dots, s \\
&v_i \geq 0 \qquad i = 1, \dots, s \\
&u_0 \geq \varepsilon
\end{aligned}
$$

Similarly for recognizing the impact of deleting l(th) output ($l \in \{1, 2, \dots, s\}$) on DMU_o (with increasing return to scale), the following model is presented:

Model 3- Preservation IRTS for deleting l(th) output

$$
\begin{aligned}
&\text{Min} \quad u_l \\
&\text{s.t.} \quad \sum_{i=1}^{m} v_i\, x_{io} = 1 \\
&\sum_{r=1}^{s} u_r\, y_{ro} + u_0 = 1 \\
&\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + u_0 \le 0 \qquad j = 1, \dots, n \\
&u_r \ge 0 \qquad\qquad r = 1, \dots, s \\
&v_i \ge 0 \qquad\qquad i = 1, \dots, s \\
&u_0 \ge \varepsilon
\end{aligned}
$$

Models (2) and (3) are extended for decreasing return to scale or constant return to scale by changing the constraint $u_0 \ge \varepsilon$ by $-u_0 \ge \varepsilon$ or $u_0 = 0$.

Theorem 1. Suppose that $DMU_o \in \partial T_v$ (DMU_o is at least weak efficient) and it has increasing return to scale.

(a) If in the optimal solution of Model 2, $v_k^* \ne 0$ then with deleting k(th) input , DMU_o doesn't belong to ∂T_v (the new ∂T_v). Otherwise (if $v_k^* = 0$) DMU_o is remained on ∂T_v (the new ∂T_v) and also it has increasing return to scale.

(b) If in the optimal solution of model3, $u_l^* \ne 0$ then with deleting l(th) input , DMU_o doesn't belong to ∂T_v (the new ∂T_v). Otherwise (if $u_l^* = 0$) DMU_o is remained on ∂T_v (the new ∂T_v) and also it has increasing return to scale.

Proof.

(a) Suppose that $v_k^* \neq 0$ and with deleting k(th) input, DMU_o has still remained on ∂T_v. It means that in the Model 1 the optimal value of objective function is equal to one after deleting the variable v_k. Suppose this optimal solution is $(v_1^*, \dots, v_{k-1}^*, v_{k+1}^*, \dots, v_m^*, u_1^*, \dots, u_s^*, u_0^*)$ with $u_0^* > 0$. This optimal solution with $v_k = 0$ is a feasible and also optimal solution for Model 2 which is in contradiction with $v_k^* \neq 0$. Otherwise (if $v_k^* = 0$), the optimal solution of Model 2 is also optimal with optimal objective function of one for Model 1 after deleting the variable v_k. Therefore with deleting k(th) input, DMU_o is remained on ∂T_v (The new ∂T_v) and also it has increasing return to scale.

(b) Proof is similar to part (a).

3.2 Deleting multiple inputs and (or) outputs

Suppose that DMU_o is efficient and it has increasing return to scale. Now, in this section, the impact of deleting multiple inputs and (or) multiple outputs on return to scale of DMU_o is surveyed. Suppose that the impact of deleting inputs $i_1, i_2, \dots, i_k$; $0 \leq k \leq m-1$ and outputs $r_1, r_2, \dots, r_l$; $0 \leq l \leq s-1$ on the efficiency and RTS status of DMU_o is considered. As mentioned in the previous section, in deleting one input or one output if there is an optimal solution with $v_{i_p}^* = 0$ for each p = 1, …, k and $u_{r_q}^* = 0$ for each q = 1, …, l then with deleting all of these k inputs and l outputs, DMU_o is still remained on ∂T_v (the new ∂T_v). On the other hand for preservation status of return to scale it is considered one of the constraints $u_0 \geq \varepsilon$ or $-u_0 \geq \varepsilon$ or $u_0 = 0$ with respect to the initial return to scale of DMU_o. For this reason the following model is suggested.

Model 4- Preservation IRTS for deleting inputs $i_1, i_2, \dots, i_k$; and outputs $r_1, r_2, \dots, r_l$;

$$f^* = \text{Min} \sum_{p=1}^{k} v_{i_p} + \sum_{q=1}^{l} u_{r_q}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{ro} + u_0 = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 \qquad j = 1, \dots, n$$

$$u_r \ge 0 \qquad r = 1, \dots, s$$
$$v_i \ge 0 \qquad i = 1, \dots, s$$
$$u_0 \ge \varepsilon$$

Theorem 2. Suppose that $DMU_o \in \partial T_v$ (DMU_o is at least weak efficient) and it has increasing return to scale. If in the optimal solution of model 4, $f^* \neq 0$ then with deleting all of the inputs $i_1, i_2, \dots, i_k$, $0 \le k \le m-1$ and the outputs $r_1, r_2, \dots, r_l$, $0 \le l \le s-1$, DMU_o doesn't belong to ∂T_v (The new ∂T_v). Otherwise (if $f^* = 0$) DMU_o is remained on ∂T_v (The new ∂T_v) and also it has increasing return to scale.

Proof.

Suppose that $f^* \neq 0$ and with deleting all of the inputs $i_1, i_2, \dots, i_k$, $0 \le k \le m-1$ and the outputs $r_1, r_2, \dots, r_l$, $0 \le l \le s-1$, DMU_o has still remained on ∂T_v. It means that in the (Model 1) the optimal value of objective function is equal to one after deleting the variables $v_{i_1}, v_{i_2}, \dots, v_{i_k}$ and $u_{r_1}, u_{r_2}, \dots, u_{r_l}$. This optimal solution of (Model 1) with $u_0^* > 0$ and $v_{i_1} = v_{i_2} = \dots = v_{i_k} = u_{r_1}, = u_{r_2} = \dots = u_{r_l} = 0$ is a feasible and also optimal solution for Model 2 which is in contradiction with $f^* \neq 0$. Otherwise (if $f^* = 0$), the optimal solution of Model 4 is also optimal with optimal objective function of one for Model 1 after deleting the variables $v_{i_1}, v_{i_2}, \dots, v_{i_k}$ and $u_{r_1}, u_{r_2}, \dots, u_{r_l}$. Therefore with deleting inputs $i_1, i_2, \dots, i_k$, $0 \le k \le m-1$ and the outputs $r_1, r_2, \dots, r_l$, $0 \le l \le s-1$, DMU_o is remained on ∂T_v (the new ∂T_v) and also it has increasing return to scale.

Lemma. In Model 4 if $\sum_{i=1}^{k} v_i^* + \sum_{r=1}^{l} u_r^* = 0$ then in Model 2 and Model 3, $v_i^* = 0$ and $u_r^* = 0$ respectively but the opposite is not always true (Example 2 and Table 6).

4. NUMERICAL EXAMPLES

(a) Example 1

Now the presented models in this paper are used for the data of Table 3 related to 12 DMUs with two inputs and two outputs. These data have been extracted from Cooper *et al*. (2007) with a little changing. Decision making units A, B, J, k and L are evaluated efficient through BCC model (Model 1). Also by using the definition of return to scale (definition of Banker *et al.* (1992)), it is concluded that A has constant return to scale, B has increasing return to scale and the units L, K and J have decreasing return to scale.

The results of deleting the inputs and outputs on the efficiency status and return to scale of these efficient DMUs are presented in Table 4 and Table 5. The results in Table 5 are achieved by the definition of return to scale (definition of Banker *et al*. (1992)) and the results of Table 4 are obtained through solving Models 2, 3 and 4 by using the GAMS software.

(b) Example 2

In this example the presented models in this paper are used for 28 DMUs with three inputs and three outputs. These data are extracted from Charnes *et al*. (1989).

The results of deleting the inputs and outputs on the status of the efficient DMUs for stability of return to scale are presented in Table 6. The results in Table 6 are obtained through solving Models 2, 3 and 4 by using the GAMS software.

TABLE 3: DMU's data (Cooper *et al.* (2007) with a little changing)

Hospital/DMU	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Doctors (I1)	19	19	25	27	22	55	33	31	30	50	53	38
Nurses (I2)	120	131	160	168	158	255	235	206	244	268	306	284
Outpatients (O1)	170	150	160	180	94	230	220	152	190	250	260	250
Inpatients (O2)	197	50	55	72	66	90	88	80	100	100	147	120

TABLE 4: The results of deleting the inputs and outputs

	DMU_o	Model (2)		Model (3)		Model (4)			
		Min v_1	Min v_2	Min u_1	Min u_2	Min v_1+u_1	Min v_1+u_2	Min v_2+u_1	Min v_2+u_2
CRTS	A	0	0	0	0	0	0	0	0
IRTS	B	≠0	0	0	0	≠0	≠0	≠0	0
DRTS	J	0	≠0	≠0	0	≠0	0	≠0	≠0
	K	0	0	≠0	0	≠0	0	≠0	0
	L	≠0	0	≠0	0	≠0	≠0	≠0	0

TABLE 5: Type of RTS before and after deleting

DMU_s(BCC-efficient)	**A**	**B**	**J**	**K**	**L**
RTS before deleting	CRTS*	IRTS**	DRTS***	DRTS	DRTS
RTS after deleting I_1	CRTS	Inefficient	DRTS	DRTS	Inefficient
RTS after deleting I_2	CRTS	IRTS	Inefficient	DRTS	DRTS
RTS after deleting O_1	CRTS	IRTS	Inefficient	Inefficient	Inefficient
RTS after deleting O_2	CRTS	IRTS	DRTS	DRTS	DRTS
RTS after deleting I_1+O_1	CRTS	Inefficient	Inefficient	Inefficient	Inefficient
RTS after deleting I_1+O_2	CRTS	Inefficient	DRTS	DRTS	Inefficient
RTS after deleting I_2+O_1	CRTS	Inefficient	Inefficient	Inefficient	Inefficient
RTS after deleting I_2+O_2	CRTS	IRTS	Inefficient	DRTS	DRTS

**CRTS=constant return to scale, **IRTS=increasing return to scale, ***DRTS=decreasing return to scale*

TABLE 6: The results of deleting the inputs and outputs

	DMU_o	Model (2)			Model (3)			Model (4)	
		Min V_1	Min V_2	Min V_3	Min U_1	Min U_2	Min U_3	Min $V_1+V_2+U_1+U_2$	Min $V_1+V_2+U_2+U_3$
CRTS	DMU_1	0	0	0	0	0	0	≠0	≠0
	DMU_8	0	0	0	0	≠0	0	≠0	0
	DMU_9	≠0	0	0	0	0	0	≠0	≠0
	DMU_{13}	0	0	0	0	0	0	0	0
	DMU_{21}	≠0	≠0	≠0	≠0	0	0	≠0	≠0
	DMU_{24}	≠0	0	≠0	≠0	0	0	≠0	≠0
	DMU_{26}	0	0	≠0	0	0	0	0	0
IRTS	DMU_{23}	0	≠0	0	0	0	0	≠0	≠0
	DMU_{25}	≠0	0	≠0	0	0	0	≠0	≠0
	DMU_{27}	0	0	0	0	0	0	≠0	≠0

5. CONCLUSION

One of the most important issues in DEA is sensitivity analysis and stability of return to scale (RTS) with changing the inputs and outputs. Deleting one or multiple inputs or outputs in DEA can change the efficiency and RTS of some DMUs. In this paper our aim is to investigate the impact of deleting one or multiple inputs and (or) outputs on RTS and efficiency of DMUs. Some models have been presented to preserving the status of RTS. Finally the presented models are utilized through two examples.

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